International Journal of Computational and Experimental Science and ENgineering (IJCESEN)

Vol. 11-No.3 (2025) pp. 6934-6939

http://www.ijcesen.com

ISSN: 2149-9144

Research Article

Generalized F-contraction Mapping in G-Metric Spaces and Some Fixed Point Results

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Article Info:

DOI: 10.22399/ijcesen.3850
Received : 20 February 2025
Accepted : 01 March 2025

Abstract:

In this research paper, we define some new notions of generalized F-contraction of type (L) and type (J) in G-metric spaces. By using these notions we define some fixed point theorems. We also provided an example to justify our results.

Keywords

G-metric space
Fixed point
Generalized F-contraction
MSC for 2020
47H10
54H25

1. Introduction

A metric space is a nonempty set □ with a map □ of two variables that enables us to determine the distance between two points. We must compute more than merely the distance between two locations in higher mathematics. In advanced mathematics, we have to distinguish not only between integers and vectors, but also between sequences and functions. Finding a good idea of a metric space may be done in a number of ways in this discipline. Different generalizations of a metric space have been examined by numerous eminent mathematicians. The concept of G-metric space was introduced by Sims and Mustafa (2006), who also offered the following important extension of a metric space.

Definition 1.1.[3] Let P be a avoid set and that $G: P^3 \to R$ is a map that possesses the following characteristics:

$$(i)\ \ G(p,q,r) = 0\ if\ p = q = r;$$

$$(ii)\ \ 0 < G\,(p,p,q)\ whenever\ p \neq q;$$

$$(iii)\ \ G(p,p,q) \leq G\,(p,q,r), q \neq r;$$

$$(iv)\ G\,(p,q,r) = G\,(p,r,q) = G\,(q,p,r) = G\,(r,p,q) = G\,(q,r,p) = G\,(r,q,p);$$

$$(v)\ \ G\,(p,q,r) \leq G\,(p,s,s) + G\,(s,q,r),\ \forall\ p,q,r,s \in P.$$

The function G is thus referred to as a G-metric on P. G-metric space is defined as the pair (P, G).

The Banach contraction principle is a helpful finding in fixed point theory pertaining to a contraction mapping that Banach established in 1922.

Definition 1.2. [1] Let $T: P \to P$ be a self-mapping and let (P, b) be a complete metric space. For all $p, q \epsilon P$ with $p = q$, let $b\,(Tp, Tq) < b\,(p, q)$. After then, □ is referred to as a Banach contraction. In 2012, Wardowski [6] developed the fixed point theory for F-contractions and presented a new contraction, the F-contraction. In this sense, Wardowski's conclusion about the Banach contraction principle differs from the notable results of the literature. Furthermore, Piri and Kumam derived fixed point theorems of the Wardowski type in entire metric spaces [5]. Four new words were recently created by Piri and Kumam [5] in response to Dung and Hang's [2] observation: Wardowski [6] characterized the F-contraction as follows:

Definition 1.3. [6] Let (P, b) be a metric space and let $T: P \to P$ be a self mapping. Then, T is called an F-contraction on (P, b), if $\exists\ F \in \tau$ such that

$$b\,(Tp, Tq) > 0 \Longrightarrow \beta + F\,(b\,(Tp, Tq)) \le F\,(b\,(Tp, Tq))$$

for all $p, q \in P$, where τ is class of all mappings $F: (0, \infty) \to R$ such that

(F1) F is strictly increasing function, that is, $\forall\ c, d \in (0, \infty),\ if\ c < d,$ then $F(c) < F(d)$.

(F2) For every sequence $\{c_n\}$ of natural numbers, $c_n = 0$ if and only if $F\,(c_n) = -\infty$.

(F3) There exist $s \in (0,1)$ such that $c)) = 0$.

Wardowski [6] gave some examples of τ as follows:

1. $F(\varphi) = In\ \varphi$.
2. $F(\varphi) = -\frac{1}{\varphi^{\frac{1}{2}}}$.
3. $F(\varphi) = In\,(\varphi) + \varphi$.
4. $F(\varphi) = In(\varphi^2 + \varphi)$.

Definition 1.5. [7] Let (P, b) be a metric space and $ꓶ: P \to P$ be a function. ꓶ is known as F-weak contraction on (P, b), if there exists $F \in \tau$ and $\beta > 0$ such that $\forall\ p, q \in P$, $b\,(ꓶx, ꓶy) > 0$ implies that

$$\beta + F\,(b\,(ꓶp, ꓶq)) \le F\left(max\left\{b(p,q), b(ꓶp,p), b(q,ꓶq), \frac{b(p,ꓶq)+b(q,ꓶp)}{2}\right\}\right)$$

Definition 1.6. [7] Let (P, b) be a complete metric space and $ꓶ: P \to P$ be an F-weak contraction. If F or ꓶ is continuous, then ꓶ has a unique fixed point $p^* \in P$ and the sequence $\{ꓶ^n p\}$ converges to p^* for every $p \in P$, where n varies from $1\ to\ \infty$.

Dung and Hang [2] investigated the concept of generalized F-contraction and proved useful fixed point results for such kinds of functions.

Definition 1.7. [2] Let (P, b) be a metric space and $T: P \to P$ be a self mapping. Then, T is called a generalized F-contraction on (P, b), if there exist $F \in \tau$ and $\delta > 0$ such that $\forall\, p, q \in P$, $b\,(Tp, Tq) > 0$ implies that

$$\delta + F\left(b(Tp, Tq)\right) \le F\,(max\{b(p,q), b(p,Tp), b\,(q,Tq), \frac{b(p,Tq)+b(q,Tp)}{2}, \frac{b(T^2p,p)+b(T^2p,Tq)}{2}, b(T^2p,Tp), b(T^2p,q), b(T^2p,Tq)\}).$$

Subsequently, Piri and Kumam [4] replace the condition (F3) with (F3’) in the definition of F-contraction given by Wardowski [6].

$(F3')$: F is continuous on $(0, \infty)$.

They gave the notation J to denote the class of all maps $F: R_+ \to R$ which fulfill the conditions $(F1)$, $(F2)$ and $(F3')$. Piri and Kumam also proved some useful fixed point results for metric spaces. Now, the conditions $(F3)$ and $(F3')$ are not associated with each other. For example, $s \ge 1$, $F(\alpha) = \frac{-1}{\alpha^s}$, then F meet the condition $(F1)$ and $(F2)$ but it does not fulfil $(F3)$, while it fulfils the condition $(F3')$. In view of this, it is significant to observe the sequel of Wardowski [6] with the functions $F \in \tau$ rather than $F \in ꓶ$.

The goal of our paper is to propose new notions of generalized F-contraction of type (L) and type (J) in G-metric space and prove fixed point theorems for such functions.

2. Main Results

Throughout the paper, we use the following notations.

L_G is the class of all functions $F: (0, \infty) \to R$ such that

(L1) F is strictly increasing, that means $p < q$ implies that $Fp < Fq$, where p, q are positive reals.

(L2) $lim_{n\to\infty} c_n = 0$ iff $lim_{n\to\infty} F\,(c_n) = -\infty$, for every sequence $\{c_n\}$ of positive numbers .

(L3) F is continuous on $(0, \infty)$.

J_G is class of all maps $F: (0, \infty) \to R$ such that

(J1) F is strictly increasing, that means $p < q$ implies that $Fp < Fq$, where p, q are positive reals.

(J2) $lim_{n\to\infty} c_n = 0$ if and only if $lim_{n\to\infty} F(c_n) = -\infty$, for every sequence $\{c_n\}$ of positive numbers

(J3) There exists $m \in (0,1)$ such that $lim_{c\to 0^+} c^m F(c) = 0$.

Definition 2.1. Let (P, G) be a G-metric space and $T: P \to P$ be a mapping. Then, T is known as generalized F-contraction of type (L), if $\exists\ F \in L_G$ and $\lambda > 0$ such that $G\,(Tp, Tq, Tr) > 0$, then

$$\Lambda + \mathrm{F}\left(\mathrm{G}(\mathrm{Tp}, \mathrm{Tq}, \mathrm{Tr})\right) \le \mathrm{F}\left(\mathrm{S_T}(\mathrm{p}, \mathrm{q}, \mathrm{r})\right), \quad (2.1)$$

where,

$$S_T\,(p, q, r) = \{G\,(p, q, r), G\,(p, Tp, Tp), G\,(q, Tq, Tq), G\,(r, Tr, Tr)\}.$$

Definition 2.2. Let (P, G) be a G-metric space and $T: P \to P$ be a mapping. Then, T is known as generalized F-contraction of type (J), if $\exists\, F \in J_G$ and $\lambda > 0$ such that $G\,(Tp, Tq, Tr) > 0$, then

$$\lambda + F\left(G\left(Tp, Tq, Tr\right)\right) \leq F\left(S_T(p,q,r)\right),$$

Where,

$$S_T(p,q,r) = \{G(p,q,r), G(p,Tp,Tp), G(q,Tq,Tq), G(r,Tr,Tr)\}.$$

Example 2.3. Let $P = [0,3]$.
We define G on

P by $G(p,q,r) = |p - q| + |q - r| + |r - p|$.

Let $T : P \to P$ be defined as $Tp = 2$, when $p \in [0,3)$ and $Tp = \frac{1}{3}$, if $p = 5$.
Now, (P, G) is complete metric space. By choosing

$$Fp = In\, p \text{ and } \lambda = In\, \frac{1}{3},$$

we get that T is generalized F-contraction of type (L) and type (J).

Theorem 2.4. Let (P, G) be a complete G-metric space and $T : P \to P$ be a generalized F-contraction of type (L). Then, T has a unique fixed point $\mu \in P$ and the sequence $\{T^n(p_0)\}$, where $n \in N$, converges to μ for each $\mu \in p$.

Proof: Let $p_0 \in P$ and $\{p_n\}$ be the Picard sequence, i.e. $p_n = Tp_{n-1}$, where $n \in N$. If $\exists\, n \in N$ such that $p_{n+1} = p_n$, then, $Tp_n = p_n$. So, p_n is fixed point of T. Let us suppose that $p_n \neq p_{n+1}$ $\forall\, n \in N$. Then, $G(p_{n+1}, p_n, p_n) > 0$ $\forall\, n \in N$. From equation (2.1), we have

$$G(Tp_{n-1}, Tp_n, Tp_n) > 0,$$

Which implies that,

$$\begin{aligned} &\lambda + F\left(G(Tp_{n-1}, Tp_n, Tp_n)\right) \\ &\leq F\left(\left\{\begin{matrix} G(p_{n-1}, p_n, p_n), G(p_{n-1}, Tp_{n-1}, Tp_{n-1}), \\ G(p_n, Tp_n, Tp_n), G(p_n, Tp_n, TP_n) \end{matrix}\right\}\right) \\ &= F\left(\left\{\begin{matrix} G(p_{n-1}, p_n, p_n), G(p_{n-1}, p_n, p_n), G(p_n, p_{n+1}, p_{n+1}), \\ G(p_n, p_{n+1}, p_{n+1}) \end{matrix}\right\}\right) \\ &= F\left(\{G(p_{n-1}, p_n, p_n), G(p_n, p_{n+1}, p_{n+1})\}\right). \end{aligned}$$

Therefore,

$$\lambda + F\left(G(p_{n-1}, p_n, p_n)\right) \leq F\left(\{G(p_{n-1}, p_n, p_n), G(p_n, p_{n+1}, p_{n+1})\}\right) \quad (2.2)$$

If there exists $n \in N$ such that

$$\{G(p_{n-1}, p_n, p_n), G(p_n, p_{n+1}, p_{n+1})\} = G(p_n, p_{n+1}, p_{n+1})$$

From equation (2.2), we get,

$$\lambda + F\left(G(p_n, p_{n+1}, p_{n+1})\right) \leq F\left(G(p_n, p_{n+1}, p_{n+1})\right)$$

A contradiction, because $\lambda > 0$.
Therefore,

$$\{G(p_{n-1}, p_n, p_n), G(p_n, p_{n+1}, p_{n+1})\} = G(p_{n-1}, p_n, p_n) \quad \forall\, n \in N.$$

From (2.2), we obtain,

$$\lambda + F\left(G(Tp_{n-1}, Tp_n, Tp_n)\right) \leq F\left(G(p_{n-1}, p_n, p_n)\right),$$

which implies that

$$F\left(G(Tp_{n-1}, Tp_n, Tp_n)\right) \leq F\left(G(p_{n-1}, p_n, p_n)\right) - \lambda.$$

Therefore,

$$F\left(G(p_n, p_{n+1}, p_{n+1})\right) \leq F\left(G(p_{n-1}, p_n, p_n)\right) - \lambda. \quad (2.3)$$

Since, $\lambda > 0$. Thus,

$$F\left(G(p_n, p_{n+1}, p_{n+1})\right) < F\left(G(p_{n-1}, p_n, p_n)\right)$$

Using the condition of (L1), F is strictly increasing. Therefore,

$$G(p_n, p_{n+1}, p_{n+1}) < G(p_{n-1}, p_n, p_n) \;\forall\, n \in N.$$

So, $\{G(p_{n+1}, p_n, p_n)\}$ is a non negative decreasing sequence of real numbers, where $n \in N$. Thus, we conclude that $lim_{n\to\infty} G(p_{n+1}, p_n, p_n) = \alpha \geq 0$. Now we claim that $\alpha = 0$. Now let us suppose that $\alpha > 0$. Also , $\{G(p_{n+1}, p_n, p_n)\}$ is a non negative decreasing sequence of real numbers, where $n \in N$. Therefore,

$$\alpha \leq G(p_{n+1}, p_n, p_n).$$

Again, by using the assumption (L1), F is strictly increasing. Therefore,

$$F\alpha \leq M(G(p_{n+1}, p_n, p_n)).$$

Using equation (2.3), we obtain

$$\begin{aligned} F(\alpha) &\leq F\left(G(p_{n-1}, p_n, p_n)\right) - \lambda \\ &\leq G(p_{n-2}, p_{n-1}, p_{n-1}) - 2\lambda \\ &\leq G(p_{n-3}, p_{n-2}, p_{n-2}) - 3\lambda \\ &\;\;\vdots \\ &\leq G(p_0, p_1, p_1) - n\lambda. \end{aligned}$$

Therefore,

$$F(\alpha) \leq F\left(G(p_0, p_1, p_1)\right) - n\lambda \ \forall \ n \in N. \quad (2.4)$$

Also, $F(\alpha)$ is a real number and

$$lim_{n\to\infty}\left[F\left(G(p_0, p_1, p_1)\right) - n\lambda\right] = -\infty.$$

Therefore,

$$\exists \ m \in N$$

such that

$$\left[F\left(G(p_0, p_1, p_1)\right) - n\lambda\right] < F\alpha \ \forall n > m. \quad (2.5)$$

Combining (2.4) and (2.5), we get

$$F(\alpha) \leq \left[F\left(G(p_0, p_1, p_1)\right) - n\lambda\right] < F\alpha \ \forall n > m.$$

This contradiction establishes that $\alpha = 0$. Now, we have

$$G(p_n, Tp_n, Tp_n) = \left(G(p_n, p_{n+1}, p_{n+1})\right) = 0.$$

Further, we claim that $\{p_n\}_{n=1}^{\infty}$ is a Cauchy sequence. Arguing by contradiction, let us suppose that there exist $\delta > 0$ and sequences $\{p_n\}_{n=1}^{\infty}, \{c(n)\}_{n=1}^{\infty}$ such that for each $n \in N$,

$$c(n) > d(n) > n, \ G\left(p_{c(n)}, p_{d(n)}, p_{d(n)}\right) \geq \delta, \ G\left((p_{c(n)-1}, p_{d(n)}, p_{d(n)})\right) < \delta. \quad (2.6)$$

Therefore,

$$\delta \leq G\left(p_{c(n)}, p_{d(n)}, p_{d(n)}\right)$$

$$\leq G\left(p_{c(n)}, p_{c(n)-1}, p_{c(n)-1}\right) + G\left(p_{c(n)-1}, p_{d(n)}, p_{d(n)}\right)$$

$$\leq G\left(p_{c(n)}, p_{c(n)-1}, p_{c(n)-1}\right) + \in$$

$$= G\left(p_{c(n)-1}, Tp_{c(n)-1}, Tp_{c(n)-1}\right).$$

Since $\alpha = 0$, we obtain

$$G(p_n, Tp_n, Tp_n) = 0. \quad (2.7)$$

So, the inequality becomes,

$$G\left(p_{c(n)}, p_{d(n)}, p_{d(n)}\right) = \delta. \quad (2.8)$$

From (2.7) there exists $z \in N$ such that

$$G\left(p_{c(n)}, Tp_{c(n)}, Tp_{c(n)}\right) < \frac{\delta}{4} \ and \ G\left(p_{d(n)}, Tp_{d(n)}, Tp_{d(n)}\right) < \frac{\delta}{4} \ \forall \ n \geq z. \quad (2.9)$$

Now, we claim that

$$G\left(Tp_{c(n)}, Tp_{d(n)}, Tp_{d(n)}\right) = G\left(p_{c(n)+1}, p_{d(n)+1}, p_{d(n)+1}\right) > 0 \ \forall \ n \geq z. \quad (2.10)$$

Again by contradiction, let us suppose that there exists $s \geq N$, such that

$$G\left(p_{c(s)+1}, Tp_{d(s)+1}, Tp_{d(s)+1}\right) \geq 0. \quad (2.11)$$

Combining (2.6), (2.9), (2.11), we get

$$\delta \leq G\left(p_{c(s)}, p_{d(s)}, p_{d(s)}\right)$$

$$\leq G\left(p_{c(s)}, p_{c(s)+1}, p_{c(s)+1}\right) + G\left(p_{c(s)+1}, p_{d(s)}, p_{d(s)}\right)$$
$$\leq G\left(p_{c(s)}, p_{c(s)+1}, p_{c(s)+1}\right) + G\left(p_{c(s)+1}, p_{d(s)+1}, p_{d(s)+1}\right) + G\left(p_{d(s)+1}, p_{d(s)}, p_{d(s)}\right)$$

$$= \left(p_{c(s)}, Tp_{c(s)}, Tp_{c(s)}\right) + G\left(p_{c(s)+1}, p_{d(s)+1}, p_{d(s)+1}\right) + G\left(p_{d(s)}, Tp_{d(s)}, Tp_{d(s)}\right) < \frac{\delta}{4} + 0 + \frac{\delta}{4} = \frac{\delta}{2},$$

which is a contradiction and hence our supposition is wrong.
Thus, we get

$$G\left(Tp_{c(n)}, Tp_{d(n)}, Tp_{d(n)}\right) = G\left(p_{c(n)+1}, p_{d(n)+1}, p_{d(n)+1}\right) > 0 \ \forall \ n \geq z.$$

From (2.10) and assumption of the theorem, we obtain

$$\lambda + F\left(G\left(Tp_{c(n)}, Tp_{d(n)}, Tp_{d(n)}\right)\right) \leq F\left(G\left(p_{c(n)}, p_{d(n)}, Tp_{d(n)}\right)\right) > 0 \ \forall \ n \geq N. \quad (2.12)$$

From (2.3), (2.8) and (2.12), we get

$$\lambda + M(\delta) \leq M(\delta),$$

which is a contradiction. Hence, $\{P_n\}_{n=1}^{\infty}$ is a Cauchy sequence. By completeness property of (P, G) $\{p_n\}_{n=1}^{\infty}$ converges to a point μ in P. Therefore,

$$G(p_n, w, w) = 0. \quad (2.13)$$

Finally, we show that $Tw = w$. Two cases arise,

(i) $\forall \ n \in N \ \exists \ k_n \in N, k_n > k_{n-1}, k_0 = 1 \ and \ p_{k_{n+1}} = Tw.$

(ii) $\exists \ m_3 \in N, \forall \ n \geq m_3, G(Tp_n, Tw, Tw) > 0.$

In the first case

$$w = \lim_{n\to\infty} p_{k_{n+1}} = \lim_{n\to\infty} Tw = Tw.$$

In the second case, using the assumption of the Theorem 2.4, we get

$$\lambda + F\big(G(p_{n+1}, Tw, Tw)\big) = \lambda + F\big(G(p_n, Tw, Tw)\big)$$

$$\le F\left(\begin{Bmatrix} G\,(p_n, Tw, Tw), G\,(p_n, Tw, Tw), G\,(w, Tw, Tw), \\ G\,(w, Tw, Tw) \end{Bmatrix}\right).$$

For each $n \ge m_3$.

From (L3), (2.13) and taking limit when $n \to \infty$, the above inequality becomes $\lambda + M\big(G(w, Tw, Tw)\big) \le M\big(G(w, Tw, Tw)\big)$, which is a contradiction. So, our supposition is wrong. Therefore, $Tw = w$. Next, we show that T has at most one fixed point. On the contrary, we suppose that w and x are two fixed points of T, such that $Tw = w \neq x = Tx$. Now, $G(Tw, Tx, Tx) = G(w, x, x) > 0$.
From (2.1), we get

$$F\,\big(G\,(w, x, x)\big) < \lambda + F\,\big(G\,(w, x, x)\big)$$
$$= \lambda + F\,\big(G\,(Tw, Tx, Tx)\big)$$
$$\le$$
$$F\,(\{G\,(w, Tw, Tx), G\,(w, Tw, Tw), G\,(x, Tx, Tx), G\,(x, Tx, Tx)\}\,) \quad (2.14)$$
$$= F\,(\{G\,(w, x, x), G\,(w, w, w), G\,(x, x, x), G\,(x, x, x)\}\,)$$
$$= F\,(w, x, x).$$

It is a contraction. Therefore, $G(w, x, x) = 0$, that means $w = x$. This establishes that the fixed point of T is unique.

Theorem 2.5. Let $(P, G\,)$ be a complete G-metric space and $T : P \to P$ be a continuous generalized F-contraction of type (J). Then, T has a unique fixed point $\mu \in P$ and the sequence $T^n(p_0)$, where $n \in N$ converges to μ, for each $\mu \in P$.

Proof: By using identical procedure which is used in Theorem 2.4, we get

$$M\big(G(p_n, p_{n+1}, p_{n+1})\big) = M\big(G\,(Tp_{n-1}, Tp_n, Tp_n)\big)$$

$$\le M\,\big(G\,(p_{n-1}, p_n, p_n)\big) - \lambda$$

$$< M\,\big(G\,(p_{n-1}, p_n, p_n)\big).$$

Therefore,

$$G\,(p_n, Tp_n, Tp_n) = G\,(p_n, p_{n+1}, p_{n+1}) = 0.$$

As in proof of Theorem 2.4, we can prove that $\{p_n\}$ is a Cauchy sequence. Also, $(P, G\,)$ is complete metric space. Therefore, $\{p_n\}$ converges to some point $\mu \in P$. Since, T is continuous. Therefore,

$$G\,(w, Tw, Tw) = G(p_n, Tp_n, Tp_n) = G(p_n, p_{n+1}, p_{n+1}) = 0. \quad (2.15)$$

By using identical steps used in proof of Theorem 2.4, we can prove that μ is unique fixed point of T.

Example 2.6. Let $P = [0,2)$. We define G on P by

$$G\,(p, q, r) = |p - q\,| + |q - r\,| + |r - p\,|.$$

Let $T : P \to P$ be defined as $Tp = 4$, when $p \in [0,2)$ and $Tp = \frac{1}{4}$ if $p = 2$. Now $(P, G\,)$ is complete metric space. Since, T is not continuous. Therefore, T is not F-Contraction. For $p \in [0,2)$ and $q = 2$,

$$G(Tp, T2, T2) = G\left(4, \frac{1}{4}, \frac{1}{4}\right)$$

$$= \left|4 - \frac{1}{4}\right| + \left|\frac{1}{4} - \frac{1}{4}\right| + \left|\frac{1}{4} - 4\right|$$

$$= \frac{15}{2} > 0$$

Further,

$$\{G\,(p, q, r), G\,(p, Tp, Tp), G\,(q, Tq, Tq), G\,(r, Tr, Tr)\}$$
$$\ge G\,(p, Tp, Tp) +, G\,(q, Tq, Tq)$$

$$G(2,4,4) + G\left(2, \frac{1}{4}, \frac{1}{4}\right) = 4 + \frac{14}{4} = \frac{15}{2} > 0$$

Therefore,

$$G(Tp, f2, f2)$$
$$\le \frac{1}{4}\{G(p, q, r), G(p, Tp, Tp), G(q, Tq, Tq), G(r, Tr, Tr)\}\,.$$

Now, by choosing $Fp = In\,p$ and $\lambda = In\frac{1}{2}$, we obtain that T is generalized F-contraction of type (L) and type (J). Let $(P, b\,)$ and T be defined as in the above example. Since, T is not an F-weak contraction, because T is not continuous. So, Theorem 1.6 is not implemented to f on $(X, d\,)$. Also, T is a generalized F-contraction of type (L) and type (J) and $(P, b\,)$ is complete, hence Theorem 2.4 and 2.5 are implemented to T on $(P, b\,)$.

Theorem 2.7. Let $(P, G\,)$ be a complete G-metric space and $T : P \to P$ be a map such that

$$G(Tp, Tq, Tq) \le l\,G(p, q, q) + m\,G(p, Tp, Tp) + n\,G(q, Tq, Tq), \forall x, y \in X, \text{ where } l, m, n \ge 0$$

with $s + t + u < 1$. Then

(i) T has a unique fixed point $\mu \in P$.

(ii) For each $p \in P, if\ T^{n+1}p = T^n p\ \forall\, n \in N \cup \{0\}$, then $T^n\,p = w$.

Proof: It is given that $\forall\, p, q \in P$,

$$G(Tp,Tq,Tr) \le sG\,(p,q,q) + tG\,(p,Tp,Tp) + uG\,(q,Tq,Tq).$$

$$\le (s+t+u)max\{G\,(p,q,r), G\,(p,Tp,Tp), G\,(q,Tq,Tq), G\,(r,Tr,Tr)\}$$
$$\le k\{G\,(p,q,r), G\,(p,Tp,Tp), G\,(q,Tq,Tq), G\,(r,Tr,Tr)\},$$

where $k = s + t + t \in [0,1)$

If $G(Yp,Tq,Tq) > 0$, we get

$$In\frac{1}{k} + In(G\,(Tp,Tq,Tq) \le In(max\{G(p,q,r), G(p,Tp,Tp), G(q,Tq,Tq), G(r,Tr,Tr).$$

By taking $F = In(l)$ and $\lambda = In\,\frac{1}{k}$ in Theorem 2.4 and 2.5, we get the result.

Author Statements:

- **Ethical approval:** The conducted research is not related to either human or animal use.
- **Conflict of interest:** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper
- **Acknowledgement:** The authors declare that they have nobody or no-company to acknowledge.
- **Author contributions:** The authors declare that they have equal right on this paper.
- **Funding information:** The authors declare that there is no funding to be acknowledged.
- **Data availability statement:** The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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