International Journal of Computational and Experimental Science and ENgineering (IJCESEN)

Vol. 11-No.4 (2025) pp. 8374-8381
http://www.ijcesen.com

ISSN: 2149-9144

Research Article

Numerical Investigation of Fluid Effects on Heat Transfer of Nanofluids in a Plane Channel: Application to Solar Thermal Panels

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Article Info:

DOI: 10.22399/ijcesen.3506
Received : 14 July 2025
Accepted : 28 October 2025

Keywords

Laminar mixed convection
Nanofluids
Nanoparticles
Finite difference method
Thomas Algorithm (TDMA)

Abstract:

In this work, a numerical investigation is conducted to analyze laminar mixed convection of nanofluids in a two-dimensional horizontal channel. The lower wall of the channel dissipates heat at constant hot temperature, whereas the upper wall is adiabatic. The analysis considers five volume concentrations (φ) of copper nanoparticles (Cu), ranging from 0 to 0.2, suspended in common base fluids such as ethylene glycol, soybean oil, and sunflower oil. The effective thermal conductivity and dynamic viscosity of the nanofluid are estimated using the Maxwell-Garnett and Brinkman models, respectively. The set of governing equations is derived and solved numerically through a finite difference discretization. For numerical resolution, a line-by-line sweeping strategy in combination with the Thomas algorithm (TDMA) is implemented. The objective of this study is to investigate how viscosity and thermal conductivity influence both heat transfer and flow behavior, in order to determine the nanofluid that maximizes thermal performance. The outcomes are expected to contribute to enhancing the use of nanofluids in advanced thermal control systems.

1. Introduction

Conventional fluids with low thermal conductivity such as water, oils, or ethylene glycol are commonly employed in industrial applications such as pharmaceutical and chemical industries, heat exchangers, nuclear power plants, concentrated solar power plants, etc. However, their thermal performance is limited compared to that of solids [1, 2]. To augment the thermal conductivity of these fluids, solid nanoparticles are suspended in the base fluid, as the intrinsic heat conductivity of these particles significantly exceeds that of the host fluid [3,4]. In particular, incorporating metallic nanoparticles with superior thermal properties leads to a substantial enhancement in the overall thermal conductivity of the resulting nanofluid. In recent years, nanofluids have gained significant attention across various industrial applications due to their enhanced heat transfer properties [5-8]. For example, Beck et al. [9] studied the thermal conductivity of alumina nanoparticles suspended in water, ethylene glycol, mixtures thereof, showing a marked improvement in the thermal performance of the resulting nanofluids. Another study by Xu et al. [10] offers a theoretical and numerical analysis of nanofluid flow in a vertical channel. The study aims to understand the effects of mixed convection, particle size, and the Grashof number on flow characteristics. The findings reveal that mixed convection significantly influences temperature and velocity fields, and that the addition of nanoparticles enhances cooling performance. This research provides valuable insights for developing high-performance cooling systems utilizing nanofluids. Cimpean and Pop. [11] conducted a numerical simulation of mixed convective flow fully developed in an oblique channel filled with three kinds of nano-fluids: Cu-water, $Al_2\ O_3$ -water, and TiO_3 -water. Channel walls are subject to a regular heat flux, while maintaining a constant flow rate throughout the channel. The Navier-Stokes equations were resolved with the finite difference approach, whilst the computations for heat and mass transfer were executed utilizing the method of finite volume. This study explored the effects of various parameters, including mixed convection, channel inclination angle, Rayleigh number, and solid phase volume ratio, upon the flow dynamics, heat transfer, and mass transfer characteristics. The results show that increasing the solid phase volume fraction enhances the heat and mass transfer, and mixed convection effect becomes more pronounced as the channel inclination angle increases. In numerical simulations of mixed convection, a comprehensive review of the existing literature shows that numerous researchers have extensively studied and measured mixed convection phenomena [12-14].

The quality of nanofluids is influenced not only by the type of nanoparticles but also by their shape. This was investigated by Aaiza et al. [15] who examined the impacts of four different nanoparticle shapes-plates, blades, cylinders, and bricks, all with equal volume fractions-dispersed in conventional base fluids. The study analyzed how these different shapes affected the flow speed and thermal distribution of the nanofluids, revealing varied outcomes. In a related study, Kim et al. [16] surveyed the impact of brick-shaped, plate-shaped, and blade-shaped nanoparticles on thermal conductivity and stability of nanofluids containing boehmite alumina particles in water. Their findings showed that the brick-shaped nanoparticles provided the greatest suspension stability and resulted in the greatest improvement in conductivity compared to the other shapes.

The effect of magnetic fields on nanofluids have been widely studied to enhance heat and mass transfer. Das et al. [17] performed a numerical analysis of magnetohydrodynamic (MHD) flow with hybrid convection into a vertical channel full of nanofluids. The Navier-Stokes, heat, & concentration equations were numerically solved via the finite volume approach. The study thoroughly examined the impact of the Lorentz force, mixed convection, nanoparticle concentration, and Hartmann number on flow dynamics and thermal transmission. The findings demonstrate that the application of a magnetic field can improve cooling efficiency by diminishing the thermal boundary layer and augmenting turbulence. The incorporation of nanoparticles enhances the thermal conductivity of the fluid, hence increasing cooling efficiency. These findings offer significant insights for the design of high-performance heat exchangers employing nanofluids in magnetic fields. Rashidi et al. [18] performed a computational analysis of mixed convectional heat exchange of nanofluids in a channel having sinusoidal walls under the influence of an external magnetic field. The conservation equations for mass, momentum, and energy were numerically resolved via the finite volume approach. The results demonstrated that the use of an external magnetic field, alongside nanoparticles, significantly alters flow dynamics and enhances heat transfer efficiency within the channel. The sinusoidal design of the channel walls significantly influences flow dynamics and thermal performance, especially under conditions where inertia forces prevail.

In recent years, the concept of hybrid nanofluids has received increasing interest in the scientific community. These fluids are engineered by dispersing a mixture of different nanoparticles into a base fluid, with a view to enhancing thermal properties, particularly thermal conductivity [19, 20]. In a related study, Zainal et al. [21] examined the hybrid convective flow over a vertical plate under convective boundary conditions using hybrid nanofluids. Iqbal et al. [22] investigated the behavior of a Cu-Cu-water nanofluid mixture following a vertical channel, taking into account the influences of Hall current and radiant heat. Saeed et al. [23] investigated the mixed convection flow of a hybrid $H_2\ O$-Cu-Cu/$Al_2\ O_3$ nanofluid within a vertical channel exposed to radiative heat flux. Their study also examined the influence of other parameters, including Rayleigh number, Reynolds number, and nanoparticle size fraction, under convective boundary conditions. The collective findings of these studies highlight the notable influence of hybrid nanofluids and thermal radiation on flow speed and temperature distribution.

This study investigates fully developed mixed convection flow in a horizontal plane channel filled with nanofluids, where the bottom wall is heated by a constant temperature heat source. The research aims to examine the effects of varying copper nanoparticle volume fractions ($0 \leq \varphi \leq 0.2$) in different base fluids (ethylene glycol, soybean oil, and sunflower oil) on heat transfer and flow structure. The goal is to identify the base fluid that most effectively enhances heat transfer performance within the system.

2. Problem Definitions

Figure 1 depicts the physical model under investigation, consisting of a laminar nanofluid flow within a horizontal plane channel. The upper wall of the channel is supposed adiabatic, while lower wall is kept at a constantly high temperature (T_H). The flow enters the channel with a regular velocity (U_0) at an initial temperature (T_0), where T_H is greater than T_0 . The flow is characterized as two-dimensional, incompressible, while the base fluid used is Newtonian. Viscous dissipation is neglected, and the thermophysical characteristics of nanofluids are presumed to be constant, except for the density change, which is calculated using the Boussinesq approximation.

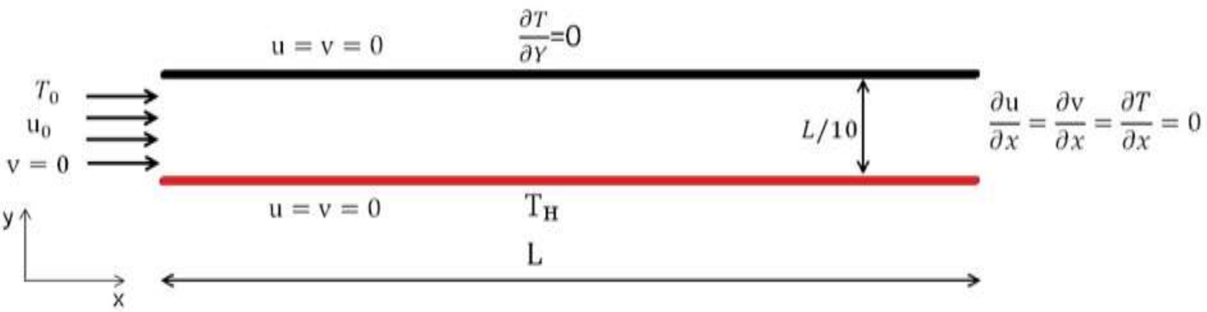


Figure 1. *The geometry of the problem studied and the boundary conditions*

3. Mathematical Models

A two-dimensional, laminar regime is assumed for both flow and heat transfer, with constant fluid properties, and the Boussinesq approximation is applied. The dimension-less formulas for the temperature (θ), stream function (ψ), and vorticity (ω) are expressed as follows:

Vorticity equation:

$$\frac{\partial \omega}{\partial \tau} + U\frac{\partial \omega}{\partial X} + V\frac{\partial \omega}{\partial Y} = \frac{1}{Re}\frac{\rho_f}{\rho_{nf}}\frac{1}{(1-\varphi)^{2.5}}\left(\frac{\partial^2 \omega}{\partial X^2} + \frac{\partial^2 \omega}{\partial Y^2}\right) - \frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f} Ri\frac{\partial \theta}{\partial X} \quad (1)$$

Energy equation:

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f}\frac{1}{Re\cdot Pr}\left[\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right] \quad (2)$$

Stream function equation:

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = \omega \quad (3)$$

The relationships between the velocity components and the stream function–vorticity formulation are given by the following equations:

velocity components:

$$U = \frac{\partial \psi}{\partial Y} \quad (4)$$

$$V = -\frac{\partial \psi}{\partial X} \quad (5)$$

Heat transfer coefficient:

$$N_u = -\left(\frac{k_{nf}}{k_f}\right)\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0} \quad (6)$$

The dimensionless variables are defined by:

$$X = \frac{x}{L},\ Y = \frac{y}{H},\ \tau = t\frac{U_0}{L},\ U = \frac{u}{U_0},\ V = \frac{v}{U_0},$$

$$\theta = \frac{(T-T_0)}{(T_C-T_0)}$$

The parameters in these equations are the Reynolds Re, Prandtl Pr, Grashof Gr, and Richardson Ri numbers and are specified by the following relationships:

$$Re = U_0.\frac{L}{\upsilon_f}\ ;\ Pr = \frac{\upsilon_f}{\alpha_f}\ ;\ Gr = \frac{g.\beta_f.\Delta T L^3}{\upsilon_f^2};\ Ri = \frac{Gr}{Re^2}$$

In this study, a single-phase approach is adopted, treating the nanofluid as a homogeneous continuous medium. It is assumed that the nanoparticles are uniformly distributed throughout the base fluid, we can therefore calculate the properties of the nanofluid using the following equations:

Density:

$$\rho_{nf} = \varphi\rho_s + (1-\varphi)\rho_f$$

Specific heat:

$$(\rho c_p)_{nf} = \varphi(\rho c_p)_s + (1-\varphi)(\rho c_p)_f$$

Coefficient of thermal expansion:

$$(\rho\beta)_{nf} = \varphi(\rho\beta)_s + (1-\varphi)(\rho\beta)_f$$

Heat diffusivity:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

The viscosity of the nanofluid has been determined by Brinkman [24]:

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}$$

The effective thermal conductivity of the nanofluid is ascertained using the Maxwell-Garnett model [25]:The thermophysical characteristics of pure fluids (ethylene glycol, glycerol, and soybean oil) and the nanoparticle (copper) are presented in Table 1 at a temperature of T = 25°C.

$$\frac{k_{nf}}{k_f} = \frac{2k_f + k_s - 2\varphi(k_f - k_s)}{2k_f + k_s + \varphi(k_f - k_s)}$$

Table 1. *Thermophysical Characteristics of Pure Fluids as well as Nanoparticles*

	Copper (Cu)	**Ethylene glycol**	**Soybean oil**	**Sunflower oil**
Cp (J/kg.k)	385	2380	1946	2138.5
ρ (kg/m^3)	8933	1115	921.5	918.15
k (W/m.k)	400	0.251	0.1535	0.161
α (m^2/s)x10^{-7}	1163.1	0.946	0.856	0.820
β (k^{-1})x10^{-5}	1.67	65	74.676	66.4
Viscosity (Pa.s)	-	0.0161	0.0505	0.0528
Pr	-	152.64	640.21	701.30
Boiling Point (°C)	-	197.4	232-300	285

Table 2. *Average Nusselt number comparison between our simulation results and those reported by Angirasa [26]*

Richardson number, Ri	**Nu_{moy}, [16]**	**Nu_{moy}, Present work**	**Error (%)**
0.1	19.3	18.62	3.52
1.0	17.7	17.12	3.27

4. Numerical Methods

The finite difference approach was employed to discretize equations (1) to (5). Diffusive terms were estimated with a centered difference scheme, while an upwind scheme was adopted for convective terms. For time integration of equations (1) and (2), the Crank–Nicolson method was employed, and the TDMA was used to obtain numerical solutions. The discretized form of equation (3) for the stream function was solved iteratively at each point using the Gauss method.

The numerical code was implemented in Fortran and its accuracy was validated by comparing results from the forced convection ventilation of a square enclosure. In this benchmark case, the vertical wall on the left side is subjected to a fixed temperature, while the other walls are assumed to be thermally insulated. The air enters through an opening located at the lower-left corner and exits through another at the top, directly across from the inlet. The average Nusselt number along the heated surface was computed and matched against the results reported by Angirasa [26]. The relative difference between the computed and reference values of the average Nusselt number does not exceed 4% for both Richardson number cases, as shown in Table 2. Thus, we can confidently conclude that our numerical code is accurate and suitable for further simulations.

5. Results and Discussions

To investigate the effect of the heat transfer fluid and copper nanoparticle volume fraction on heat transfer within the plane channel, numerical simulations were conducted using three different fluids: ethylene glycol, soybean oil, and sunflower oil, corresponding to Prandtl numbers of Pr = 152.64, 640.21, and 701.30, respectively. Simulations were performed for various nanoparticle volume fractions (φ = 0.00, 0.05, 0.10, 0.15, and 0.20).

5.1 Isotherm

The temperature distribution for each combination of volume fraction and Prandtl number is presented in Figure 2, which displays isotherm contours illustrating temperature stratification. Notably, high temperatures are confined to narrow regions near the hot wall, corresponding to the thermal boundary layer thickness, which is significantly influenced by the volume fraction and Prandtl number values. Further away from the hot wall, temperature gradients are relatively low. The results indicate that for high volume fractions (φ = 0.2) and large Prandtl numbers (Pr = 701.30, such as in sunflower oil), temperatures are elevated and tightly confined to narrow regions near the hot wall. Conversely, as the volume fraction and Prandtl number decrease, the thermal boundary layer thickness increases.

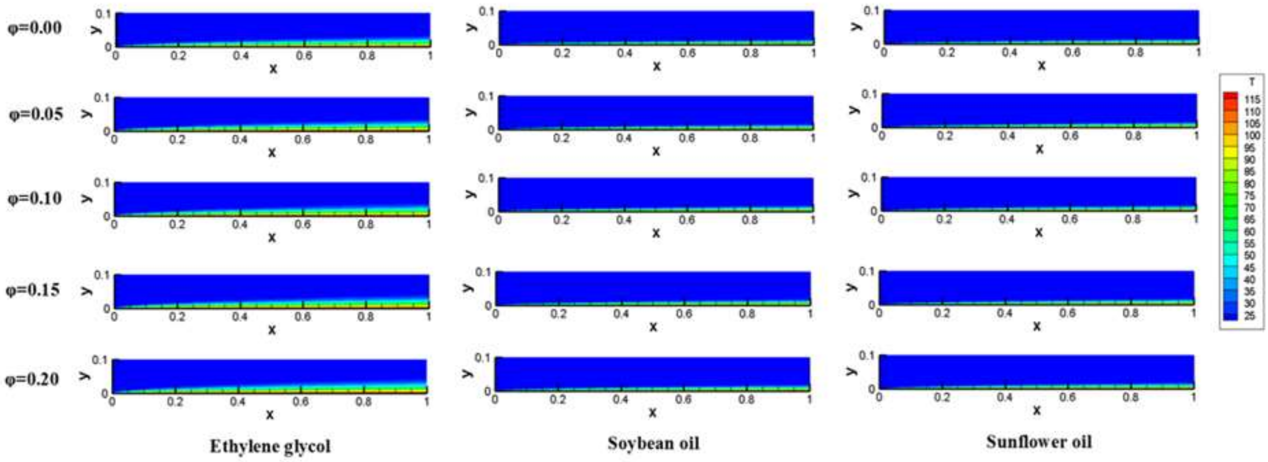


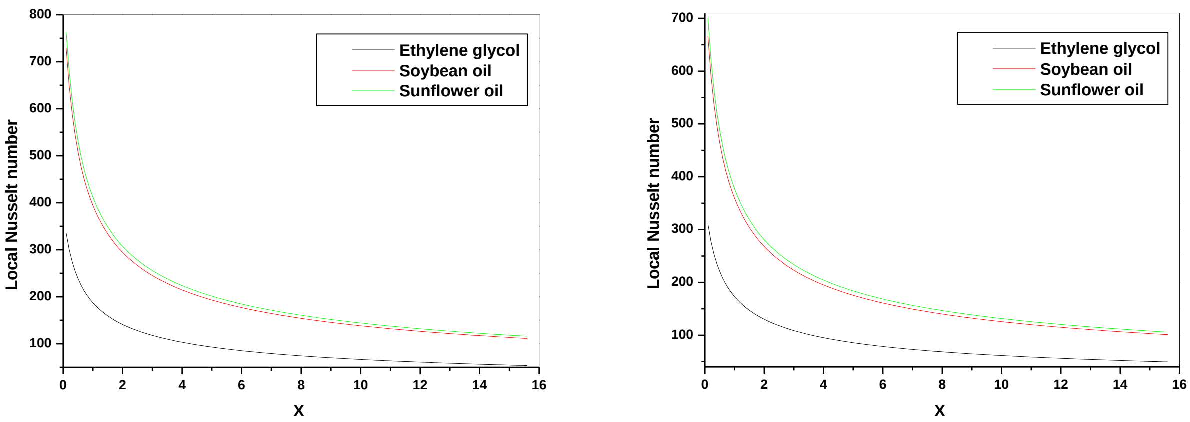
Figure 2. *Isotherms for different fluids and nanoparticle volume fractions, while Gr= 10^6 and Re=10^3*

5.2 Heat transfer

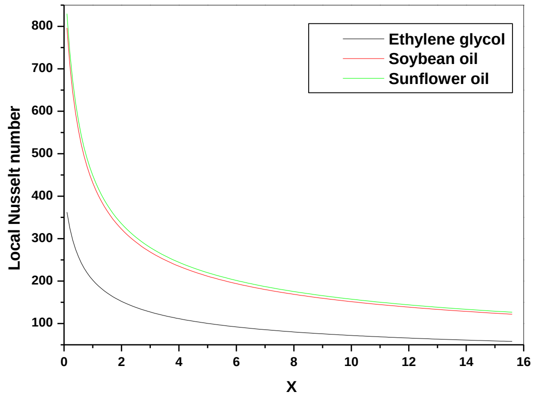
A detailed description of the heat transfer behavior is presented in Figure 3, which illustrates the distribution of the local Nusselt number along the heated wall based on the Prandtl number and various volume fractions. Initially, it can be seen that the highest heat transfer is located at the left edge of the heated surface. The data clearly shows that the local Nusselt number rises as the volume fraction increases, suggesting improved thermal performance. Furthermore, thermal exchange at the hot wall is notably enhanced when sunflower oil (Pr = 701.30) is used, owing to its superior thermal conductivity. On the other hand, ethylene glycol (Pr = 152.64) exhibits the lowest heat transfer among the tested fluids, resulting in the smallest local Nusselt number values.

Figure 4 demonstrates the total transfer of heat characteristics at the hot walls, revealing that the mean Nusselt number improves as the size fraction of the nanoparticles increases. In the absence of nanoparticles, the heat transfer of a base fluid is constrained by its low conductivity, resulting in lower temperature gradients adjacent to the heated wall and consequently lower average Nusselt numbers. As the concentration of nanoparticles increases, the nanofluid's thermal conductivity improves, leading to more pronounced temperature gradients and a corresponding boost in the average Nusselt number.

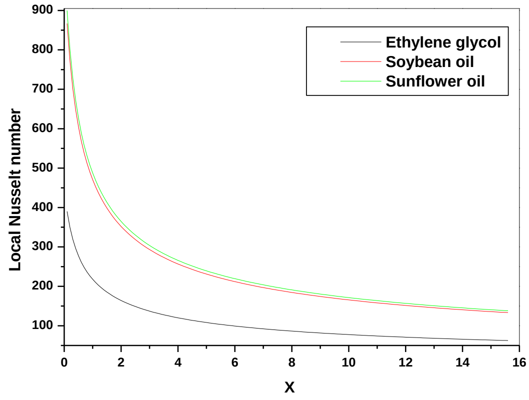
It's also clear to see that the differences in the average Nusselt number relative to the Prandtl number suggest that greater heat dissipation occurs as the Prandtl number rises. Lastly, the most significant average Nusselt number is observed when sunflower oil (Pr = 701.30) acts as the heat transfer medium, confirming the trend shown in Figure 3.



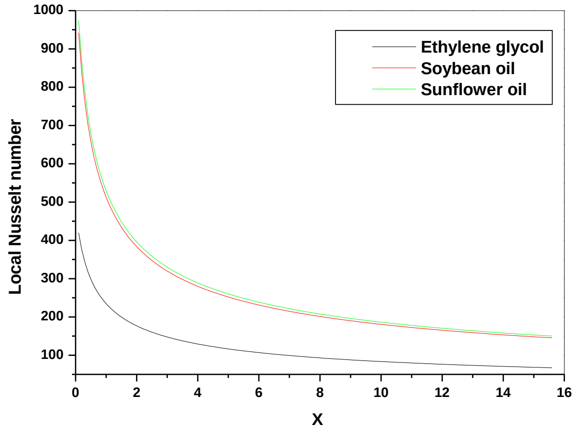
(a) *(b)*



(c)



(d)



(e)

Figure 3. *Distribution of the local Nusselt number along the heated wall for various fluids at various nanoparticle volume fractions: (a) φ=0.00, (b) φ=0.05, (c) φ=0.10, (d) φ=0.15, (e) φ=0.20*

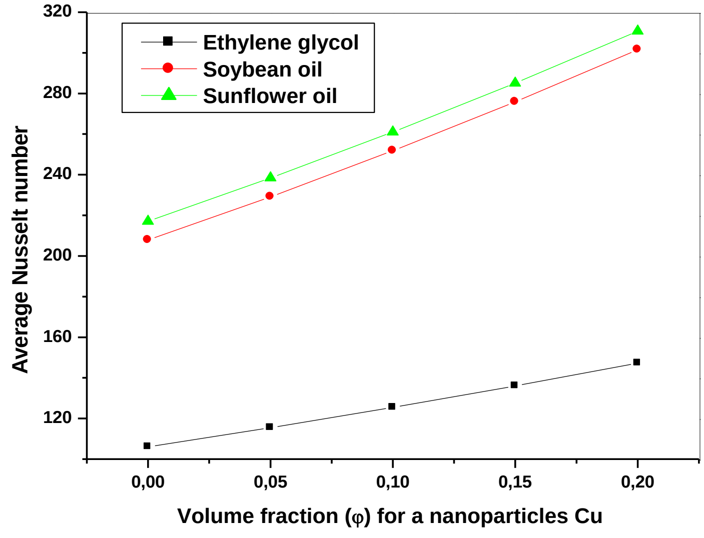


Figure 4. *Variation in the average Nusselt number throughout the heated wall for various fluids and volume fractions*

4. Conclusions

This study focuses on the numerical investigation of heat transfer by laminar mixed convection of nanofluids in a horizontal plane channel, using various fluids and a nanoparticle volume fraction (φ) of Copper (Cu) ranging from 0 to 0.2. The primary objectives of this research were to optimize heat transfer by examining the influence of different factors, including fluid type and nanoparticle volume fraction. The principal findings of this study can be encapsulated as follows:

1) An increase in the nanoparticle volume fraction (φ) was found to significantly enhance the Nusselt number, indicating improved heat transfer efficiency and more effective thermal interaction between the fluid and the heated wall.
2) Among the fluids studied, sunflower oil exhibited the highest heat transfer, followed by soybean oil, with ethylene glycol showing the lowest heat transfer.
3) In summary, the thermo-physical properties of the heat transfer fluid, along with the nanoparticle volume fraction, significantly influence heat transfer in the horizontal plane channel.

Author Statements:

- **Ethical approval:** The conducted research is not related to either human or animal use.
- **Conflict of interest:** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper
- **Acknowledgement:** The authors declare that they have nobody or no-company to acknowledge.
- **Author contributions:** The authors declare that they have equal right on this paper.
- **Funding information:** The authors declare that there is no funding to be acknowledged.
- **Data availability statement:** The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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